“Max” Utility Function

A consumer has the utility function $u(x_1, x_2) = \max\{x_1, x_2\}$. What is the Marshallian demand function for the two goods by the consumer and what are the indirect utility functions, the Hicksian demands, and the expenditure function?

Solution

To find the optimal demand for this individual, we must consider that if it happens that both x_1 and x_2 are different from 0 at the same time, money would be wasted since the utility function only takes the maximum value between the two goods. Therefore, the solution will be a corner solution and will depend on relative prices.

$$x_1^m = \begin{cases} \frac{m}{p_1} & \text{if } p_1 < p_2 \\ 0 & \text{if } p_1 > p_2 \end{cases}$$

$$x_2^m = \begin{cases} 0 & \text{if } p_1 < p_2 \\ \frac{m}{p_2} & \text{if } p_1 > p_2 \end{cases}$$

Therefore, what this individual does is consume only the cheaper good. The last case we need to consider is when $p_1 = p_2$. In that situation, both the basket $(0, \frac{m}{p_2})$ and the basket $(\frac{m}{p_1}, 0)$ are optimal.
The indirect utility function will also be split:

$$V = \begin{cases} \max\{\frac{m}{p_1}, 0\} = \frac{m}{p_1} & \text{if } p_1 < p_2 \\ \\ \max\{0, \frac{m}{p_2}\} = \frac{m}{p_2} & \text{if } p_1 > p_2 \end{cases}$$

And lastly, in the case where $p_1 = p_2$, the indirect utility function can take the value of m/p_1 or m/p_2.
To find the Hicksian demands, we can first find the expenditure function by inverting the indirect utility function. We first assume that $p_1 < p_2$, then the function takes the following form:

$$v = m/p_1$$

Inverting:

$$m = vp_1$$

$$e = \bar{u}p_1$$

Assuming the other case, we have the following expenditure function:

$$e = \bar{u}p_2$$

Combining these:

$$e = \begin{cases} \bar{u}p_1 & \text{if } p_1 < p_2 \\ \bar{u}p_2 & \text{if } p_1 > p_2 \end{cases}$$

And in the case where $p_1 = p_2$, the function can take either of the two forms.
Finally, for the Hicksian demands, we use the Shepherd's lemma. We assume that we are in the case where $p_1 < p_2$:

$$\frac{\partial e}{\partial p_1} = \bar{u} = x_1^h$$

$$\frac{\partial e}{\partial p_2} = 0 = x_2^h$$

Similarly, assuming the other case:

$$\frac{\partial e}{\partial p_1} = 0 = x_1^h$$

$$\frac{\partial e}{\partial p_2} = \bar{u} = x_2^h$$

Therefore, we have the following Hicksian demands:

$$x_1^h = \begin{cases} \bar{u} & \text{if } p_1 < p_2 \\ 0 & \text{if } p_1 > p_2 \end{cases}$$

$$x_2^h = \begin{cases} 0 & \text{if } p_1 < p_2 \\ \bar{u} & \text{if } p_1 > p_2 \end{cases}$$

And finally, in the case where $p_1 = p_2$, we can have either of these two Hicksian demands: $(\bar{u}, 0)$ or $(0, \bar{u})$.